$$W=\int_{q_1(o)}^{q_1(t)}(-c_1\dot{q}_1-k_1q_1+T_1)dq_1+\int_{q_2(0)}^{q_2(t)}(-c_2\dot{q}_2-k_2q_2+T)dq_2+\cdots$$

where the integrals are line integrals in the $\dot{q}_1$, $q_1$ phase space; thus the work done is path-dependent. The solution to the second example appears to be correct, indicating that Bailey again used virtual work, not work, in determing the solution.

We agree with Bailey that there is some confusion on this subject in the recent literature. For example, Goldstein[3] has an incorrect derivation of Lagrange's equations for non-conservative systems using a principle similar to that of Ref. 1 and Lanczos apparently chooses not even to discuss the issue in his treatise.[4] However, Whittaker in 1904 gave a complete discussion (see Ch. IX of Ref. 2).

### References

[1]Bailey, C.D., "Application of Hamilton's Law of Varying Action," *AIAA Journal,* Vol. 13, Sept. 1975, pp. 1154-1157.

[2]Whittaker, E.T., *Analytical Dynamics,* Dover, N.Y., 1944 (4th Edition).

[3]Goldstein, H., *Classical Mechanics,* Addison-Wesley, 1950, pp. 38-39.

[4]Lanczos, C., *The Variational Principles of Mechanics,* Toronto, 1970 (4th Edition).

## Reply by Author to A. E. Bryson Jr.

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IN the last paragraph of his comment, Professor Bryson has made, in our humble opinion, the understatement of the century. He agrees with us that there is some confusion on this subject; but, he then restricts his comment to the recent literature. He points out Goldstein[1] as being incorrect; he mentions Lanczos[2]; he doesn't mention Osgood[3]; and finally, he bases his conclusions on Whittaker.[4] Who, pray tell, are we to believe? The confusion is not recent. It has been with us for a very long time and, without doubt, will continue for years to come. This was brought home to us by the statement of an internationally recognized authority who said, "I do not like energy methods because almost every writer has a different treatment." Progress in our work was delayed almost a year because we thought Goldstein to be correct. Now Professor Bryson believes that our theory is "basically incorrect."

Hamilton did not furnish a variational principle in the sense that we are taught variational principles in the variational calculus, and we make no claim of any variational principle. Hamilton furnished the Law of Varying Action for the motion of matter in time-space.[5,6] The authority for this is in Hamilton's own words[5]; but, nowhere have we found direct analytical solutions to time-dependent, nonconservative systems from energy considerations alone, which Hamilton implied could be achieved.[6]

Our results are correct because our theory is correct; but, a definitive paper on this subject is yet to be written. The reasons for this are set forth in Ref. 7 and Ref. 8. A copy of Ref. 8 is available from either the NASA Langley Research Center or from the author.

We wish to express our appreciation to Professor Bryson for writing. If our theory appears incorrect, it is because our concepts are different.[7,8] We define work in the same way that Professor Bryson finds it defined in Whittaker; and, if we should ever desire to calculate the work explicitly, we would calculate it as indicated by Professor Bryson. There is no inconsistency in its use as presented in our published papers[9-12] for reasons which we will present, "...at another time in another place."[13]

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[1]Goldstein, H., *Classical Mechanics,* Addison-Wesley, Reading, Mass. 1950.

[2]Lanczos, C., *The Variational Principles of Mechanics,* Toronto, 1970 (4th ed).

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[6]Hamilton, W. R., "Second Essay on a General Method in Dynamics," *Philosophical Transactions of the Royal Society of London,* Vol. 125, 1835, pp. 95-144.

[7]Bailey, C. D., "A New Look at Hamilton's Principle,"- *Foundations of Physics,* Vol. 5, No. 3, Sept. 1975, pp. 433-451.

[8]Bailey, C.D., "Application of the General Energy Equation: A Unified Approach to Mechanics," Final Report, NASA Grant NGR 36-008-197, Aug. 1975.

[9]Bailey, C. D., "The Method of Ritz Applied to the Equation of Hamilton," *Computer Methods in Applied Mechanics and Engineering,* Vol. 7, 1976, pp. 235-247.

[10]Bailey, C. D., "Application of Hamilton's Law of Varying Action," *AIAA Journal,* Vol. 13, Sept. 1975, pp. 1154-1157.

[11]Bailey, C.D., "Exact and Direct Analytical Solutions to Vibrating Systems with Discontinuities," *Journal of Sound and Vibration,* Vol. 44, Jan. 1976, pp. 15-25.

[12]Bailey, C. D., "Hamilton, Ritz and Elastodynamics," ASME-*Journal of Applied Mechanics,* Paper 76-APM-R.

[13]Smith, D. R. and Smith, D. V., Jr., "Reply by Authors to S. F. Felszeghy and C. D. Bailey," *AIAA Journal,* Vol. 13, Nov. 1975, p. 1540.

## Comment on "Practical Aspect of the Generalized Inverse of a Matrix"

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Hassig[1] obviously did not read the extensive references he cited on generalized inverse. The uniqueness of the generalized inverse was stated in the very beginning of Penrose (Hassig's Ref. 1). The very first equations of Greville (Hassig's Ref. 2) give Hassig's main results, Eq. (11) and (12). It is also well-known that when the solution to Hassig's Eq. (1) is not unique, the generalized inverse gives the minimum norm solution. Hassig also misses the basic concept and usefulness of generalized inverse by unduly restricting himself to matrices of maximal rank ("linearly independent" in Hassig's somewhat misleading terminology). The article by Greville is meant to be an elementary exposition on generalized inverse and is easy to read. Also, most modern engineering tests on control and estimation theory have a brief treatise on the subject.

### Reference

[1]Hassig, H. J., "Practical Aspect of the Generalized Inverse of a Matrix," *AIAA Journal,* Vol. 13, Nov. 1975, pp. 1530-1531.

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